ANGULAR FACTOR DETERMINATION FOR A RADIATIVE CONVEYOR DRYING SYSTEM WITH DIFFUSE AND SPECULAR REFLECTION

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A method is given for determining the angular radiation intensity factors for thermal drying volumes with cylindrical forces.

Analytical methods and relationships are available for determining the angular radiation intensity factors (ARIF) for specular reflection [1-3]; on the other hand, a real surface usually has a combination of specular and diffuse reflection coefficients $R = r^d + r^s$. If the diffusely reflected flux is substantially nonuniform, which occurs in many practical cases, it becomes difficult to calculate the radiated heat transfer. One reason is that it is not correct to use reciprocity relations in such calculations, while it is also necessary to perform fourfold integration over the surfaces in order to determine the angular intensity factors (AIF). If one can determine ARIF on the basis of both components, one can sometimes simplify the radiative heat-transfer calculations while retaining acceptable accuracy.

Consider a closed volume bounded by gray planar walls with a relatively high reflectivity R (Fig. 1a). The conveyor belt carrying the components is assumed to act as an attenuating plane with an attenuation factor $K_3 = K_1K_2$; the method of determining this has been described [7] on the basis of the light stimulation principles of [5]. The radiation sources within the volume take the form of finite cylinders and radiate diffusely. The medium within the volume is diathermic.

We determined the ARIF from the source 1 to the part 3. The total energy falling on the part from the sources is made up of the energy directly from the sources and the energy arising from multiple reflections from the screens 2, 4, and 5; we follow [1-4] to put

$$Q_{13} = E_1F_1\varphi_{13}^0 + E_1F_1\varphi_{13}^s + E_1F_1\varphi_{13}^d = E_1F_1\Phi_{13}, \tag{1}$$

where $\Phi_{13} = \varphi_{13}^0 + \varphi_{13}^s + \varphi_{13}^d$.

To determine φ_{13}^0 we use the formula for the AIF from an infinitely small area on a finite cylinder [1]. We use the reciprocity conditions as in [6, 7] to obtain an expression for the AIF from a series of finite cylinders at a finite but arbitrary area. We take l as the definitive dimension of the sources to get

$$\varphi_{13}^0 = \frac{\varkappa H}{\pi D n} \sum_{i=1}^{n} \int_{-\delta/2}^{\delta/2} \int_0^L \frac{f^0(x,\ y)\,dxdy}{\sqrt{N_i^2 + H^2}}\,, \tag{2}$$

where

$$N_i = A + (i-1)\,S - x;$$

$$f^0(x,\ y) = \sum_{j=1}^{2} \left\{ \frac{1}{\pi V} \operatorname{arc\,tg}\left(\frac{Z_j}{\sqrt{V^2-1}}\right) + \frac{Z_j}{\pi}\left[\frac{A_j - 2V}{V\sqrt{A_jB_j}} \times \right.\right.$$
$$\left.\left. \times \operatorname{arc\,tg}\left(\sqrt{\frac{A_j(V-1)}{B_j(V+1)}}\right) - \frac{1}{V}\operatorname{arc\,tg}\left(\sqrt{\frac{V-1}{V+1}}\right)\right]\right\}. \tag{3}$$

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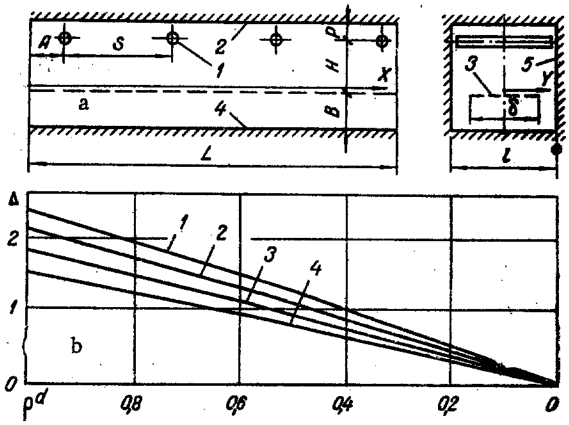

Fig. 1. a) Scheme for chamber; b) percent error curves for (8) with R of: 1) 1.0; 2) 0.8; 3) 0.6; 4) 0.4.

Here

$$Z_1=\frac{1-2y}{D};\quad Z_2=\frac{1+2y}{D};\quad C=\sqrt{H^2+N_i^2};$$

$$V=\frac{2C}{D};\quad A_j=(1+V)^2+Z_j^2;\quad B_j=(1-V)^2+Z_j^2.$$

The expression for φ_{13}^S is derived similarly; here the geometrical parameters are determined by mapping methods as in [1, 6], and then

$$\varphi_{13}^s=\frac{\varkappa}{\pi Dn}\sum_{i=1}^{n}\Big[R\rho^s(2P+H)\int\limits_{-\delta/2}^{\delta/2}\int\limits_{0}^{L}\frac{f_1^s(x,\ y)\,dx\cdot dy}{C_1}+$$

$$+R^2(\rho^s)^2K_1^2K_2^2(3H+2B+2P)\int\limits_{-\delta/2}^{\delta/2}\int\limits_{0}^{L}\frac{f_2^s(x,\ y)\,dxdy}{C_2}+$$

$$+R^3(\rho^s)^3K_1^2K_2^2(3H+2B+4P)\int\limits_{-\delta/2}^{\delta/2}\int\limits_{0}^{L}\frac{f_3^s(x,\ y)\,dxdy}{C_3}+$$

$$+R^2(\rho^s)^2K_1^2K_2^2(2B+H)\,\delta\int\limits_{0}^{L}\frac{f_4^s(x)\,dx}{C_4}+R^2(\rho^s)^2K_1^2K_2^2\,(2P+2B+H)\,\delta\int\limits_{0}^{L}\frac{f_5^s(x)\,dx}{C_5},\qquad(4)$$

where

$$f_{4,5}^s(x)=\frac{1}{\pi V}\operatorname{arc\,tg}\left(\frac{Z_0}{\sqrt{V^2-1}}\right)+\frac{Z_0}{\pi}\left[\frac{(A-2V)}{V\sqrt{AB}}\times\right.$$

$$\left.\times\operatorname{arc\,tg}\left(\sqrt{\frac{A(V-1)}{B(V+1)}}\right)-\frac{1}{V}\operatorname{arc\,tg}\sqrt{\frac{V-1}{V+1}}\right],\quad Z_0=\frac{1}{D}.\qquad(5)$$

Here the first, second, and third terms incorporate, respectively, the singly, doubly, and triply specularly reflected contributions from the upper plane of the component. The fourth and fifth terms incorporate the irradiation of the lower plane.

In (4) we have discarded terms from the fourth onwards, since these contribute only about 0.5% to the total AIF. The functions $f_n^S(x,\ y)$, where n = 1, 2, 3, 4, 5, are given by

(3) and r, respectively,

$$C_1 = \sqrt{(H+2P)^2 + N_i^2}, \quad C_2 = \sqrt{(3H+2B+2P)^2 + N_i^2}.$$
$$C_3 = \sqrt{(3H+2B+4P)^2 + N_i^2}, \quad C_4 = \sqrt{(2B+H)^2 + N_i^2},$$
$$C_5 = \sqrt{(2P+2B+H)^2 + N_i^2}.$$

In some cases, analytical determination of f_4^S (x, y) and f_5^S (x, y) is difficult on account of the complex configurations of the conveyor; then it is better to derive these functions by experiments using light simulation. Our experiment showed that the energy density at the lower plane of a component is virtually uniformly distributed for $B \geq 0.4$ [6] in many cases, which simplifies (4) by reducing the number of integrations in the last two terms.

If we assume that the density of the diffusely reflected energy is substantially variable only along the X axis, we can use an expression for the AIF from an infinitely narrow strip of finite length on the final surface [1], which enables us in determining φ_{13}^d to transfer from fourfold integration over the two surfaces to twofold integration over one surface. Numerical estimates show that the resulting error in φ_{13}^d does not exceed 5%. Figure 1b shows the effects of this error on the overall error of the ARIF in relation to ρ^d. Thus we put

$$\varphi_{13}^d = \delta \int_0^L \frac{\bar{E}_{23}^d(x)\, dF_2 \varphi_{dF_2-F_3}}{E_1 F_1} + \delta \int_0^L \frac{\bar{E}_{43}^d(x)\, dF_4 \varphi_{dF_4-F_3}}{E_1 F_1}, \tag{6}$$

where $\bar{E}_{23}^d(x)$ and $\bar{E}_{43}^d(x)$ are, respectively, the mean diffusely reflected energy fluxes at screens 2 and 4, the integration being over the Y axis.

We use the approach employed in deriving (4) to get

$$\bar{E}_{23}^d(x) = PR\rho^d \sum_{i=1}^{n} \int_{-0,5}^{0,5} \frac{f_6^d(x, y)\, dy}{C_6};$$
$$\bar{E}_{43}^d(x) = R\rho^d K_1 K_2 (H+B) \sum_{i=1}^{n} \int_{-0,5}^{0,5} \frac{f_7^d(x, y)\, dy}{C_7}, \tag{7}$$

where the angular factors $f_6^d(x, y)$ and $f_7^d(x, y)$ are defined by (3), with $C_6 = \sqrt{P^2 + N_i^2}$, $C_7 = \sqrt{(H+B)^2 + N_i^2}$.

We substitute (7) into (6) to get after several steps that

$$\varphi_{13}^d = \frac{\delta R \rho^d}{\pi D n} \sum_{i=1}^{n} \left[P \int_0^L \int_{-0,5}^{0,5} \frac{f_6^d(x, y)\, \varphi_{dF_2-F_3}\, dx dy}{C_6} + K_1 K_2 (B+H) \int_0^L \int_{-0,5}^{0,5} \frac{f_7^d(x, y)\, \varphi_{dF_4-F_3}\, dx dy}{C_7} \right], \tag{8}$$

where $\varphi_{dF_{2,4}-F_3}$ may be determined from a formula derived by using an expression given in [1]:

$$\varphi_{dF_{2,4}-F_3} = \frac{1}{\pi B_1} \left[\sqrt{1+B_1^2}\, \mathrm{arctg} \left(\frac{A_1}{\sqrt{1+B_1^2}} \right) - \mathrm{arctg}\, A_1 + \right.$$
$$\left. + \frac{A_1 B_1}{\sqrt{1+A_1^2}}\, \mathrm{arctg} \left(\frac{B_1}{\sqrt{1+A_1^2}} \right) \right] + \frac{1}{\pi B_1} \left[\sqrt{1+B_1^2}\, \mathrm{arctg} \left(\frac{A_2}{\sqrt{1+B_1^2}} \right) - \right.$$
$$\left. - \mathrm{arctg}\, A_2 + \frac{A_2 B_1}{\sqrt{1+A_2^2}}\, \mathrm{arctg} \left(\frac{B_1}{\sqrt{1+A_2^2}} \right) \right]. \tag{9}$$

Here for $\varphi_{dF_2-F_3}$ $B_1 = 1/(P + H)$; $A_1 = x/(P + H)$; $A_2 = (L - x)/(P + H)$; while for $\varphi_{dF_4-F_3}$ $B_1 = 1/B$; $A_1 = x/B$; $A_2 = L - x/B$.

We sum (2), (4), and (8) to finally get that

$$\Phi_{13} = \frac{1{,}18\varkappa}{\pi D n} \sum_{i=1}^{n} \Bigg[H \int_{-\delta/2}^{\delta/2} \int_{0}^{L} \frac{f^0(x,\ y)\,dxdy}{C} + \rho^s R\,(2P + H) \times$$

$$\times \int_{-\delta/2}^{\delta/2} \int_{0}^{L} \frac{f_1^s(x,\ y)\,dxdy}{C_1} + R^2 (\rho^s)^2 K_3^2 (3H + 2B + 2P) \times$$

$$\times \int_{-\delta/2}^{\delta/2} \int_{0}^{L} \frac{f_2^s(x,\ y)\,dx \cdot dy}{C_2} + R_3 (\rho^s)^3 K_3^2 (3H + 2B + 4P) \int_{-\delta/2}^{\delta/2} \int_{0}^{L} \frac{f_3^s(x,\ y)\,dx\,dy}{C_3} +$$

$$+ R\rho^s K_1^2 K_2 \delta\,(2B + H) \int_{0}^{L} \frac{f_4^s(x)\,dx}{C_4} + R^2 (\rho^s)^2 K_1^2 K_2 \delta\,(2P + 2B + H) \int_{0}^{L} \frac{f_5^s(x)\,dx}{C_5} +$$

$$+ R\rho^d \delta P \int_{0}^{L} \int_{-0{,}5}^{0{,}5} \frac{f_6^d(x,\ y)\,\varphi_{dF_2-F_3}\,dx \cdot dy}{C_6} + R\rho^d K_3 (B + H)\,\delta \int_{0}^{L} \int_{-0{,}5}^{0{,}5} \frac{f_7^d(x,\ y)\,\varphi_{dF_4-F_3}\,dx \cdot dy}{C_7}. \qquad (10)$$

Here the factor 1.18 incorporates within ±2.5% the fraction of the energy incident on the component by reflections from the end screens 5; this quantity was determined from measurements with a light model for $0.1 \le H \le 0.8$.

We evaluated the overall accuracy of (10) by comparing the ARIF derived from a light model with numerical calculations by a Minsk-22 computer; the discrepancy did not exceed 5%.

To make (10) convenient to use in engineering calculations, we constructed nomograms of Z type with binary scale (Figs. 2-5), which enable one to determine $\Phi_{13} = f(R, \rho^d, S, H, K)$; the overall result for the ARIF Φ_{13} is determined by simple summation of the values for the individual components. The nomograms are designed for the working dimensions most commonly encountered in practice, and they can readily be supplemented if the actual parameters differ substantially from those given.

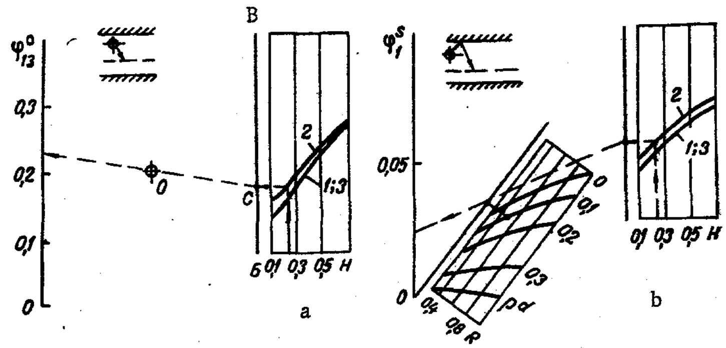

Fig. 2. Nomograms for determining: a) φ^0_{13} (1 — S = 0.4; 2 — 0.8; 3 — 1.2-1.6) for A = 0.1; B = 0.3; P = 0.1; L = 4; δ = 0.75; K_1 = 0.7; b) ARIF components φ^s_1 for single specular reflection.

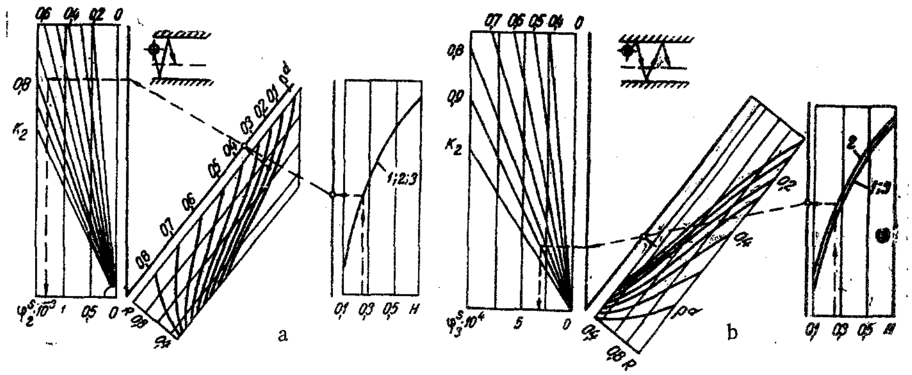

Fig. 3. Nomograms for determining ARIF components for: a) double specular reflection φ_2^s; b) triple specular reflection φ_3^s.

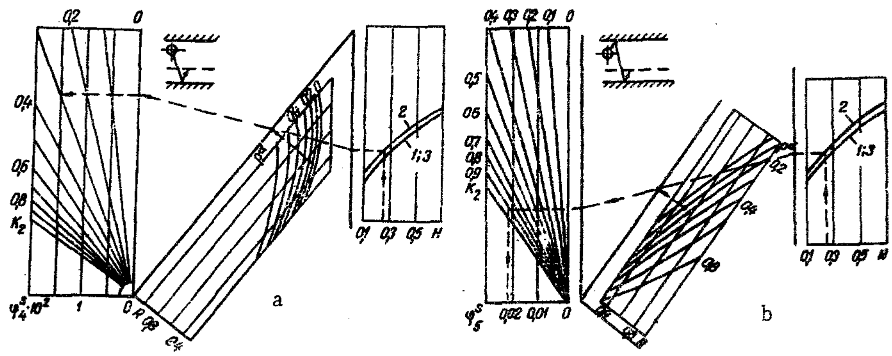

Fig. 4. Nomograms for determining ARIF components with respect to underside of irradiated surface: a) Single specular reflection φ_4^s; b) double specular reflection φ_5^s.

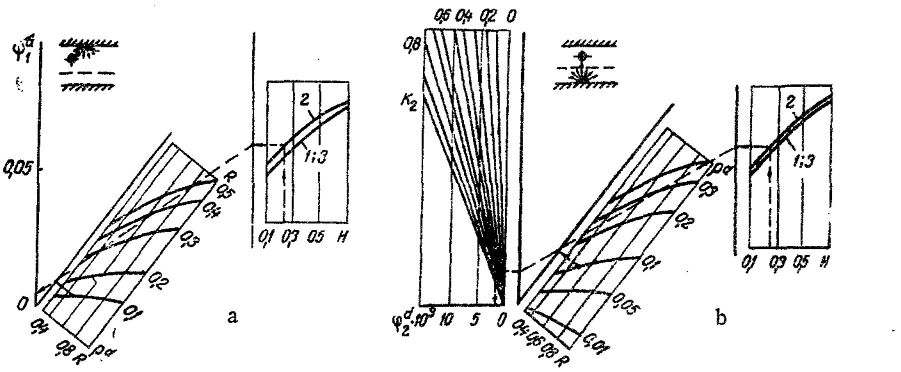

Fig. 5. Nomograms for determining ARIF components with diffuse reflection: a) from the upper screen φ_1^d; b) from the lower screen φ_2^d.

NOTATION

E, density of radiant energy; E^d, density of energy diffusely reflected from screen; F, body area; n, number of sources; K_1, coefficient incorporating energy absorption by conveyor; K_2, coefficient incorporating energy absorption by jobs; φ^0, angular irradiance factor; φ^s_{13}, component of angular resolution irradiation factors (ARIF) incorporating specular reflection; φ^d_{13}, component of ARIF incorporating diffuse reflection; Φ ARIF, $f_{1-5}(x, y)$, angular factors for radiation from a strip of infinitely small width and finite length to finite rectangle;

$\varphi_{dF_{2,4}-F_3}$, angular factors for radiation from infinitely small area to finite cylinder; ρ^s, relative specular reflectivity; ρ^d, relative diffuse reflectivity; x, conveyor filling factor.

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